# Effects of Projection Geometry on Shape Perception of 3D Cuboids by 2D Drawings 

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#### Abstract

Visual perception of 3D objects from 2D drawings is a remarkable ability of human vision and has been extensively studied．However，the effect of two ways of projection geometry，perspective and parallel projection， remains under debate．In order to investigate the effect of the projection geometries on shape／size perception， we asked 32 subjects to judge whether the 2D drawings of 3D cuboids look like a＂box＂or a＂plate＂through varying the Height／Width（H／W）ratio of the 3D cuboids．Results showed that drawings of the 3D cuboids in perspective projection were judged as box with higher probability than parallel projection．The cuboids in parallel projection needed $8.3 \%$ higher $\mathrm{H} / \mathrm{W}$ ratio to yield the same＂boxness＂as perspective projection．The results suggest that the difference in shape perception may reflect that perspective projection would enhance the perception of a larger object than parallel projection．Thus，the perceived shape／size of 3D cuboids by 2D drawing is influenced by projection geometry．


Keywords－perspective projection，parallel projection，shape／size perception

## 1．Introduction

Since the Renaissance，artists and psychologists have
been debating whether the visual perception of two－dimensional（2D）drawings of three－dimensional （3D）objects is governed by perspective，parallel projection，others，or combination of those，and to what extent the drawings of 3D objects will look more natural and realistic to an observer．Early Greek， Roman，Byzantine，Persian，and Chinese art all provide instances of the use of parallel projection（Arnheim， 1974；Dubery \＆Willats，1983；Hagen \＆Jones，1978； Hagen，1986；Kubovy，1986）．However this widespread use of parallel projection to represent 3D scenes poses some problems for theories contending that the perception of depth in pictures follows the laws of linear perspective projection geometry，while the parallel projection drawings do not adhere to the perspective projection laws（Nichols \＆Kennedy， 1993）．

In perspective projection，sets of parallel lines seem to converge in some vanishing points（usually in a single point），resulting in non－uniform foreshortening of object．The object size appears smaller as distance from the center of projection increases and becomes larger as that distance decreases，providing observer a more realistic representation of an object．Therefore the shape of an object is rarely preserved and perspective distortion is induced in perspective projection（Arnheim， 1977；Carlbom \＆Paciorek，1978；Kennedy \＆Juricevic，

2002；Mcgreevy \＆Ellis，1986；Veltman，1987）． Parallel projection corresponds to a perspective projection with an infinite focal length and no vanishing points．It primarily attempts to represent metric properties rather than realistic visual appearance of an object．In parallel projection，the lines of sight from the object to the projection plane are parallel to each other and having a fixed same direction．Objects do not appear larger or smaller as they extend closer to or away from the observer，hence the relative length and shape of object is preserved（Carlbom \＆Paciorek， 1978；Hagen \＆Elliott，1976；Nichols \＆Kennedy， 1993）．

Perspective projection assumes a finite focal length， namely，distance from the observer is variable．Since parallel projection is a perspective projection with an infinite focal length，there is no assumption of distance between an observer and an object（Hagen \＆Elliott， 1976）．In general，perspective distortion of a picture with perspective projection is stronger for wide angle and short distance．Besides，a larger perspective distortion suggests that an observed object is a larger， rather than small object（John，James，John，and Robert 1980；Nicholls \＆Kennedy，1992）．Therefore，we expected that projection geometry（e．g．，perspective or parallel projection）could affect the perceived size of a visual object．

In the present study，we examined this possibility by asking subjects to judge whether a cuboid would look as＂plate＂or＂box＂．Since large 3D objects such as buildings tend to be classified as＂box＂rather than ＂plate＂（even though the height／width ratio is similar to corresponding object that are smaller in size），we hypothesized that the proportion of＂box＂responses would be influenced by the way the object is depicted．
corrected－to－normal vision，and were naive about the purpose of the study．

## Stimuli

Visual stimuli were displayed on a LCD monitor with screen resolution of $1280 \times 800$ pixels（approximately $25.65 \times 16.15 \mathrm{deg}$ in visual angle）．Forty images of 3D cuboids as stimuli were created with perspective and parallel projection．The perspective projection stimuli were produced in three－point perspective projection；the parallel projection stimuli were projected in orthographic projection with three adjacent faces visible（Figure 1，examples of visual stimulus）．The stimulus images of cuboids were made by a commercially－available 3D rendering software，with the virtual camera elevated $30^{\circ}$ from the horizontal plane； object rotated $30^{\circ}$ from the front about the vertical axis （i．e．，azimuth angle $30^{\circ}$ ）．The viewing distance between camera and object was fixed．The volume of the cuboid stimuli was fixed and defined as 1 unit；the base of the cuboid stimuli was square whereas the ratio of height to width（H／W）varied from 0.07 to 0.45 while keeping the volume of cuboid constant（Table 1，Stimulus parameters）．The visual stimuli subtended from $1.61^{\circ} \times$ $23.05^{\circ}$ to $5.5^{\circ} \times 12.41^{\circ}$ of visual angle depending on the $\mathrm{H} / \mathrm{W}$ ratio at a viewing distance of 60 cm ．Each stimulus image was shown 5 times in random order．

## Procedure

The subjects were seated in front of the screen，and they were asked to judge whether the stimulus looked like a ＂plate＂or a＂box＂by pressing one of the two pre－set keys．The proportion of judged to be a＂box＂was recorded separately for the perspective and parallel projection conditions．

## 2．Method

## Subjects

Thirty－two undergraduates（ 12 female，with mean age $=$ $21.6, \mathrm{SD}=1.8$ ）participated．All of them had normal or

| H／W <br> Ratio | Perspective projection | Parallel projection |
| :--- | :--- | :--- |
|  |  |  |
| 0.13 |  |  |
| 0.21 |  |  |
| 0.29 |  |  |

Figure 1．Examples of visual stimulus（cuboids）

| H／W Ratio | Volume | VA－height $\left({ }^{\circ}\right)$ | VA－width $\left({ }^{\circ}\right)$ |
| :---: | :---: | :---: | :---: |
| 0.07 | 1.0 | 1.61 | 23.05 |
| 0.09 | 1.0 | 1.91 | 21.20 |
| 0.11 | 1.0 | 2.18 | 19.83 |
| 0.13 | 1.0 | 2.44 | 18.75 |
| 0.15 | 1.0 | 2.68 | 17.88 |
| 0.17 | 1.0 | 2.92 | 17.15 |
| 0.19 | 1.0 | 3.14 | 16.52 |
| 0.21 | 1.0 | 3.36 | 15.98 |
| 0.23 | 1.0 | 3.57 | 15.51 |
| 0.25 | 1.0 | 3.77 | 15.08 |
| 0.27 | 1.0 | 3.97 | 14.70 |
| 0.29 | 1.0 | 4.16 | 14.35 |


| 0.31 | 1.0 | 4.35 | 14.04 |
| :--- | :--- | :--- | :--- |
| 0.33 | 1.0 | 4.54 | 13.75 |
| 0.35 | 1.0 | 4.72 | 13.48 |
| 0.37 | 1.0 | 4.90 | 13.23 |
| 0.39 | 1.0 | 5.07 | 13.00 |
| 0.41 | 1.0 | 5.24 | 12.79 |
| 0.43 | 1.0 | 5.41 | 12.59 |
| 0.45 | 1.0 | 5.58 | 12.40 |

Table 1．Stimulus parameters

## 3．Result

We fitted psychometric functions for the proportion of ＂box＂response and H／W ratio separately for the perspective projection and parallel projection（Figure 2）． The proportion of＂box＂responses increased with the $\mathrm{H} / \mathrm{W}$ ratio increasing．And with the $\mathrm{H} / \mathrm{W}$ ratio increasing，the proportion of＂box＂response in perspective projection was always higher than that in parallel projection，which implied the drawings in perspective projection tend to be perceived as＂box＂ more than in parallel projection．


Figure 2．The proportion of＂box＂responses as a function of $\mathrm{H} / \mathrm{W}$ ratio of the cuboid．Psychometric functions are fitted for each projection geometry condition．

The results were subjected to a two－way analysis of variance（ANOVA）for projection geometry （perspective and parallel projection）and $\mathrm{H} / \mathrm{W}$ ratio（20）， and $p<.05$ level of significance was used in statistical tests．The ANOVA analysis showed that the main effect for projection geometry（perspective versus
parallel projection）was significant $(F(1,31)=16.9, p$ $<.001$ ），which indicate that perspective and parallel projection have significant different effect on shape perception of＂box＂or＂plate＂．The H／W ratio also showed a significant effect $(F(19,589)=138, p<.001)$ ， and the interaction between the way of projection and $\mathrm{H} / \mathrm{W}$ ratio was also significant $(F(19,589)=1.7, p$ $<.05$ ）．As the $\mathrm{H} / \mathrm{W}$ ratio varying from 0.23 to 0.31 ，the proportion of＂box＂response was changing significantly different between drawings in perspective and parallel projection（post－hoc multiple comparison； $p<.05$ ）．

Then，we estimated the point of subjective equality （PSE）for the plate／box judgment，that is the $\mathrm{H} / \mathrm{W}$ ratio to be judged as＂box＂and＂plate＂at equal probability， for each subject and averaged them．Individual PSEs showed that parallel projection need higher $\mathrm{H} / \mathrm{W}$ ratio to be perceived as＂box＂than perspective projection． The average PSE of perspective projection（0．22）was significantly smaller than that of parallel projection （0．24）（Figure 3，paired t－test，$t(31)=4.8, p<.001$ ）， which indicated that the cuboids depicted in perspective projection was perceived to be＂box＂at a smaller $\mathrm{H} / \mathrm{W}$ ratio than parallel projection．Figure 4 shows that the ＂ambiguous box／plate＂stimuli in perspective projection （0．22）and parallel projection（0．24）．The magnitude of shape perception difference was $8.3 \%$ in terms of $\mathrm{H} / \mathrm{W}$ ratio．These results suggested that the cuboids depicted in perspective projection tended to be seen more frequently as＂box＂rather than＂plate＂compared to those with parallel projection．


Figure 3．Averaged PSEs in the perspective projection
and parallel projection conditions．

| Perspective projection | Parallel projection |
| :--- | :--- |
|  |  |

Figure 4．Ambiguous＂box＂or＂plate＂in perspective and parallel projection．

## 4．Discussion

The subjects chose＂box＂more frequently for drawings in perspective projection than for parallel projection． The higher ratio of height to width of the cuboids，the more frequently the object in perspective projection was judged to look like a＂box＂．In order to be perceived as the same＂boxness＂as the visual object with perspective projection，the visual object with parallel projection need to be $8.3 \%$ higher in $\mathrm{H} / \mathrm{W}$ ratio． Those results indicated that for 2D drawings of 3D objects depicted may be influenced by projection geometry．

Some researchers（e．g．，Busey，Brady，Cutting， 1990；Farber \＆Rosinski，1978；Hans \＆Frederick， 1986；Pirenne，1970；Kubovy，1986；Goldstein，1987； Rosinski \＆Farber，1980；Yang \＆Kubovy，1999） proposed that seeing the perspective distortion of drawing allows observers to compensate for these transformations of pictorial space while parallel projection not．That is，observers perceive the object of pictorial space as if they were viewing from the center of projection，while parallel projection，with an infinite focal length and no central of projection，sets of all parallel lines，prevented such compensation．The two ways of geometry projection provide different levels of cues for shape and size／distance of viewed objects． Although how exactly these differences in terms of visual cues would influence the shape and size
perception of depicted objects in 2D drawings，the present results showed that the perceived shape of 3D cuboids by 2D drawing is influenced by projection geometry．

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